# Robust face recognition under the polar coordinate system 

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#### Abstract

In this paper, we propose a novel method for face recognition which uses the polar coordinate system instead of the conventional cartesian coordinate system. Among the central area of a face, we select a point as a pole and make a polar image of a face by evenly sampling pixels in each direction of 360 degrees around the pole. The polar coordinate system delineates near-pole area more vividly than the area far from the pole. Therefore, a polar face image can achieve more vivid representation of the important central facial regions compared to the conventional cartesian face image. To cope with a small amount of rotation and illumination change in a frontal face image, we suggest a method based on vertical symmetry of a face. In addition, we also deal with a scaling problem by using a correlationbased method. Experimental results show that the proposed methods enhance classification performance in the frontal face recognition problem.


Keywords: Face recognition, polar coordinate system, rotation, scaling, illumination.

## 1. Introduction

Numerous methods have been developed for face recognition in computer vision and pattern recognition community in the last few decades [1] [2] [3] [4]. Among them, many methods are based on 2D cartesian face image and this can be further divided into holistic and local feature based methods. The Eigenface [5], Fisherface [6], ICA (Independent Component Analysis) [7] and null space methods [8] [9] are the successful representatives of the holistic approach and many further improvements to these methods are still being made. In typical holistic feature based methods, each pixel of an image is regarded as an original input variable and these are projected into a lower dimensional space by using a dimensionality reduction techniques to extract important features for face recognition.

However, not all of the pixels in a face image are uniformly important in extracting the discriminant features. In [10], a pixel selection method was investigated from a face image by measuring the discriminative power of each pixel for face recognition. Much of the selected pixels in [10] were in the central part of the face image which is in consistent with the results in psychophysical reports that the eyebrows, eyes, nose, and mouth are the most helpful in facial identification [11]. The polar coordinate system naturally fits to this idea because in a polar coordinate
system, the near-pole area is more vividly represented while the area far from the pole is more coarsely represented.

The polar coordinate system is used in iris recognition [12] and object recognition [13]. In [12], iris texture is converted into a polar iris image which is a rectangular image containing iris texture represented in a polar coordinate system. In object recognition [13], object in the cartesian coordinate is mapped into the polar-log coordinate, and the closed profile of the object is transformed into an one dimensional curve. However, until now, most face recognition methods extract features from images in the cartesian coordinate system and in our knowldege, no work has been done for face recognition using the polar coordinate system.

In this paper, we address a novel method for face recognition which uses a polar coordinate system instead of the conventional cartesian coordinate system to improve classification performance. In a face, important regions such as eyes, nose and mouth are concentrated on the central part of a face and polar image can achieve more vivid representation of the central part while the periphery such as hair and beard is more coarsely represented. Among the central area of a face, we select a point as a pole and make a polar image of a face by evenly sampling pixels in each direction of 360 degrees around the pole. If the number of pixels are equal in two coordinate systems, we can get more accurate recognition rate by using polar coordinate system because the polar coordinate of a face image can achieve more vivid representation of important facial regions compared to the conventional cartesian coordinate system.

The paper is organized as follows. In section 2, the polar coordinate transform for face recognition is introduced. In addition, the methods to cope with the unfavorable conditions such as illumination, rotation and scaling variations are also introduced. The classification performances for Yale and Yale-B databases are compared with the conventional cartesian coordinate system in Section 3 and Section 4 completes the paper with a discussion and the conclusions.

## 2. Methods

### 2.1 Transformation to a polar coordinate system

In transforming cartesian image to a polar image, among the central area of a face, we select a point as a pole and make a polar image of a face by evenly sampling pixels in each direction of 360 degrees around the pole.


Fig. 1
THE ORIGINAL FACE IMAGE (A) AND THE TRANSFORMATION OF CARTESIAN TO POLAR IMAGE WITH RADIUS 40 (B).

In a face, important regions such as eyes, nose and mouth are concentrated on the central part of a face and it is assumed that these feature points play more important roles in discriminating individuals than the other areas. The polar coordinate system naturally achieves this purpose because in a polar coordinate system, the near-pole area is more vividly represented than the area far from the pole. Therefore, it is expected that the classification performance of polar images will be higher than that of cartesian images.

Fig.1(a) shows a face image in $80 \times 100$ Yale database and Fig.1(b) shows the corresponding pole and orientation of the polar coordinate system with the radius of 40 pixels. In the transformation, the location of a pole is crucial and in our experiments, we manually marked the location of the tip of a nose in each image and the average location was used as the pole. The resultant location was $(x, y)=(40,65)$ for Yale database and $(x, y)=(42,50)$ for YaleB database. The bicubic interpolation is used for the transformation and the intensity of the points outside the cartesian image is set to 0 (black).

### 2.2 Illumination compensation by symmetry

The frontal image of a face is almost symmetrical about the vertical axis. In this paper, this property of symmetry is used to reduce the effect of the varying lighting conditions of a frontal face image. If we transform a cartesian image to a polar image, the polar frontal face image is symmetrical along two axes $\left(\theta_{0}=90^{\circ}\right.$ or $\left.270^{\circ}\right)$. To reduce the effect of illumination change, we make the average intensities of the two symmetrical lines with angle $\theta$ and $\theta_{s}\left(=2 \theta_{0}-\theta\right)$ equal. This is obtained by firstly computing the average pixel values along the line with a specific angle $\theta$ and the corresponding symmetric line with angle $\theta_{s}$. Then the difference $\delta$ between the two values are computed. Finally, $\delta / 2$ is added to the pixel values for the angle $\theta$ and the same value is subtracted from the pixel values for the symmetric angle $\theta_{s}$. This


Fig. 2
A POLAR IMAGE WITH RADIUS OF 70 PIXELS (A) AND THE RESULT OF ILLUMINATION COMPENSATION (B)
procedure is described in (1).

$$
\begin{align*}
& \delta=\left(\sum_{i=1}^{r} P(i, \theta)-\sum_{i=1}^{r} P\left(i, 2 \theta_{0}-\theta\right)\right) / r  \tag{1}\\
& P^{\prime}(:, \theta)=P(:, \theta)-\delta / 2 \\
& P^{\prime}\left(:, 2 \theta_{0}-\theta\right)=P\left(:, 2 \theta_{0}-\theta\right)+\delta / 2
\end{align*}
$$

Here, $P$ is an image in a polar coordinate system, $P^{\prime}$ is the output polar image with illumination compensation and $r$ is the radius of the polar image.

Fig. 2(a) shows the polar image of Fig. 1 with radius of 70 pixels. The image is $70 \times 360$ pixels whose row is the distance from the pole in pixel and column corresponds to angle in degree. The black area in the image corresponds to the outside of the original image. Fig. 2(b) is the result of illumination compensation of Fig. 2(a). By the illumination compensation method in (1), we can reduce the intensity imbalance between the left and right side of Fig. 1(a) and increase the extent of symmetry.

### 2.3 Coping with rotations

As shown in the previous subsection, we can reduce the effect of varying illumination by the property of symmetry and this can be further applied to obtain a rotation robust polar face image. A rotation around the pole in a cartesian coordinate system corresponds to a circular shift in a polar coordinate system. By using the property of symmetry, we can estimate the amount of shift in the polar image and the rotation can be compensated. This is enabled by the simple correlation method along the symmetrical axes. However, if the left and the right sides of a face image are significantly different in intensity as in Fig. 1, this method is prone to errors. Therefore, in this paper, we propose to firstly compensate illumination by the method described in the previous subsection and then use a correlation method to estimate the degree of rotation.

The detailed algorithm is shown below as Algorithm 1.
Algorithm 1: Estimation of amount of rotation

1) Polar image: Transform the given image to a polar coordinate system.
2) Illumination compensation: Use (1) to recover illumination variations.
3) For $\theta=-\alpha$ to $\alpha$ degree,
a) Circular shift the polar image by $\theta$. The resultant image is $P$.
b) Correlation coefficient: Dissect the polar image into 4 pieces along the angle
$Q_{1}=P(:, 1: 90), Q_{2}=P(:, 180: 91)$
$Q_{3}=P(:, 181: 270), Q_{4}=P(:, 360: 271)$
and calculate the sum of correlation coefficients $c(\theta)$.
$c(\theta)=\operatorname{corrcf}\left(Q_{1}, Q_{2}\right)+\operatorname{corrcf}\left(Q_{3}, Q_{4}\right)$. Here, $\operatorname{corrcf}(A, B)$ is the correlation coefficient between the matrices $A$ and $B$.
end
4) Find the maximum value of $c(\theta)$ among various $\theta$ 's and the corresponding rotation angle $\theta^{*}$. The value $\theta^{*}$ is the estimation of rotation angle.

We have observed that the algorithm is invariant approximately up to $30^{\circ}$ of rotation from the upright position in various illumination conditions. Note that in the proposed algorithm, illumination compensation is applied firstly and then correlation method is applied afterwards. In a severe condition, like EM (expectation and maximization) algorithm, Steps 2 to 4 can iteratively be applied for better estimation.

### 2.4 Coping with scaling variations

In real applications, small size variation is inherent in face detection algorithms and a good face recognition algorithm should be invariant to the size variation. Assuming that the rotation and illumination are correctly compensated, in this paper, we use the correlation based method to cope with scaling variations.

A reference scale image is generated for each person and then for various scaled target images, we calculate the correlation coefficient between the reference image and the target image. The scale factor of the target image with the highest correlation coefficient is estimated as the output scaling factor. This procedure is described below in Algorithm 2.

[^0]a) Resize the target image with the scaling factor $s$. The resultant image is $P$.
b) Compute the correlation coefficient
$c(i, s)=\operatorname{corrcf}(M(i), P)$.
end
end
3) Find the maximum value of $c(i, s)$ and the corresponding scaling factor $s^{*}$. The value $s^{*}$ is the estimation of the scaling factor.

In our experiment, we randomly scaled original image with scaling factor uniformly distributed from 0.5 to 1.5 . In the recovering process, $\beta=0.5$ and $\gamma=1.5$ was used with step size 0.01 .

## 3. Experimental Results

In this section, we perform experimental studies on Yale and YaleB databases to evaluate the performance of the proposed algorithms.

The Yale database consists of 165 images which contains 15 individuals and 11 images per each person. Original images are 100x80 pixels in size with gray color. The YaleB database contains 10 persons and we used 64 images per person with different lighting conditions. The size of images in YaleB database are $96 \times 84$ pixels with gray color.

Table 1 shows the recognition rate of original and polar images for Yale database using LDA and NLDA. The 11 -fold cross-validation (CV) was applied to polar images with $70 \times$ 360 pixels in size. In other words, out of 11 images per each person, ten were used for training and the remaining one was used for testing. This procedure was repeated 11 times to make each image be in the test set once. The 1-nearest neighborhood ( $1-\mathrm{NN}$ ) algorithm was used as a classifier.

In the table, we show the classification rate with and without histogram equalization [14] for both cartesian and polar images. The last row shows the performance of the illumination compensation method proposed in Section 2.2 after histogram equalization. The table shows that the recognition rates of polar images are better than those of cartesian images. The recognition rates reached to $99.39 \%$ and $100 \%$ using LDA and NLDA respectively when we compensate illumination by symmetry.

Table 2 shows the results of recovery from rotation for Yale and YaleB database. Each image in the databases was rotated by a fixed amount from $-20^{\circ}$ (left) to $+20^{\circ}$ (right) and Algorithm 1 in Section 2.3 was applied to check if it estimates the degree of rotation correctly. The mean absolute error (MAE) between the true and the estimated angles and the standard deviation of the estimated angle are shown in the tables. For both databases, polar images in $70 \times 360$ were used. In the table, we can see that the MAE for Yale database is below $1.8^{\circ}$ while maximum MAE for YaleB is $2.6^{\circ}$. The standard deviations for YaleB database are rather

Table 1
The recognition rate for original image and polar image for Yale database. (11-fold CV)

| Methods | Rec. Rate (\%) |  |  |
| :---: | :---: | :---: | :---: |
|  | LDA | NLDA |  |
| Cartesian | w/o. histeq | 78.78 | 87.88 |
|  | w. histeq | 92.12 | 93.33 |
| Polar | w/o. histeq | 90.91 | 95.15 |
|  | w. histeq | 92.72 | 96.97 |
|  | symm + histeq | 99.39 | 100 |

Table 2
Experimental results for rotation
(a) Yale database

| Rotation $\left({ }^{\circ}\right)$ | 0 | +3 | +5 | +10 | +15 | +20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MAE $\left({ }^{\circ}\right)$ | 1.25 | 1.28 | 1.50 | 1.72 | 1.62 | 1.58 |
| Std. dev. $\left({ }^{\circ}\right)$ | 1.46 | 1.61 | 1.83 | 2.17 | 2.08 | 1.98 |
| Rotation $\left({ }^{\circ}\right)$ |  | -3 | -5 | -10 | -15 | -20 |
| MAE $\left({ }^{\circ}\right)$ |  | 1.46 | 1.64 | 1.80 | 1.76 | 1.72 |
| Std. dev. $\left({ }^{\circ}\right)$ |  | 1.59 | 1.74 | 2.06 | 2.04 | 2.08 |

(b) YaleB database

| Rotation $\left({ }^{\circ}\right)$ | 0 | +3 | +5 | +10 | +15 | +20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MAE $\left({ }^{\circ}\right)$ | 1.53 | 1.49 | 1.59 | 1.82 | 2.11 | 2.23 |
| Std. dev. $\left({ }^{\circ}\right)$ | 3.15 | 2.96 | 3.54 | 4.88 | 6.48 | 7.02 |
| Rotation $\left({ }^{\circ}\right)$ |  | -3 | -5 | -10 | -15 | -20 |
| MAE $\left({ }^{\circ}\right)$ |  | 1.62 | 1.69 | 2.07 | 2.42 | 2.57 |
| Std. dev. $\left({ }^{\circ}\right)$ |  | 3,74 | 4.14 | 5.84 | 7.15 | 7.70 |

large because illumination variation is much larger for YaleB than Yale database.

Table 4 shows the result of scaling variation for Yale database. Polar images in $40 \times 90$ pixels were resized to the target images with scaling factors of $0.8,1.0$ and 1.2. Then Algorithm 2 in Section 2.4 was applied to check the efficiency of the algorithm. For each individual, a mean polar image was obtained in the original scale. Then, these 15 mean images were correlated with scaled versions of a target image with various scaling factors from 0.5 to 1.5 . The target image was classified as the identity with the highest correlation coefficient and the corresponding scaling factor was stored. Among 165 images, 14, 12 and 13 images were failed to be classified correctly when the scaling factor was $0.8,1.0$ and 1.2 respectively. Among the correctly classified images, the mean and the standard deviation of the estimated scaling factor were calculated and reported in the table. The errors between the true and the estimated scaling factor and the standard deviations are quite small. It shows that the scaling problem can be solved reliably with the proposed algorithm.

## 4. Conclusions

This paper addresses a novel method for face recognition which uses a polar coordinate system instead of the con-

Table 3
Experimental results for scaling for Yale database

| Scaling Factor (SF) | 0.8 | 1.0 | 1.2 |
| :---: | :---: | :---: | :---: |
| Correct Class. Rate (\%) | 91.52 | 92.73 | 92.12 |
| Mean of est. SF | 0.809 | 0.993 | 1.204 |
| Std. dev. | 0.016 | 0.020 | 0.023 |

ventional cartesian coordinate system. In a face, important regions such as eyes, nose and mouth are concentrated on the central part of a face and polar image can achieve more vivid representation of the central part while the periphery such as hair and beard is more coarsely represented. By applying conventional feature extraction methods to the polar image, the recognition rates were improved. We also proposed methods to cope with illumination variations, rotation and scaling problems. With the simple idea of symmetry and correlation, the proposed methods performed quite satisfactorily for Yale and YaleB databases.

The limitations of the proposed methods based on polar coordinate system are 1) they only apply to frontal face images, 2) they are susceptible to mis-location of the pole. As future works, we would like to tackle these problems in addition to developing methods not based on simple correlation.

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[^0]:    Algorithm 2: Estimation of scaling factor

    1) Creation of mean image: For each person $i$ in a database with fixed scale, create the mean polar image $M(i)$.
    2) Correlation coefficient:

    For each class $i=1$ to No-person,
    For each scaling factors $=\beta$ to $\gamma$

