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Robust Control of Robot Manipulator by Model Based Disturbance Attenuation

Keywords : Robot manipulators, MBDA, position control, Liapunov function, stability.

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Abstract

In this letter, a model based disturbance attenuator (MBDA) for robot manipulators is proposed and the stability of the MBDA in robot positioning problems is proved via Liapunov's direct method. This method does not require an accurate model of a robot manipulator and takes care of disturbances or modelling errors so that the plant output remains relatively unaffected by them. The output error due to the gravity or constant disturbance can be effectively eliminated by this method.

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I. Introduction

To achieve high performance in controlling robots, much research has been conducted under the assumption that the dynamics of robot systems are exactly known. But this assumption is not usually satisfied because it is very difficult to obtain an exact robot model due to its nonlinear dynamic structure and modelling uncertainties. To overcome these problems, a model based disturbance attenuator for robot manipulators is proposed and its asymptotic stability is proved in the following. It is a generalization of [1] for robot manipulators and a preliminary result without the stability analysis was presented in [2].

II. The MBDA Controller and Its Stability

In the absence of friction and disturbances, the dynamics of an n degree of freedom robot manipulator is given by the Lagrange-Euler vector equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \Re^n$ are the vectors of generalized position, velocity, and acceleration respectively, of n links, $M(q) \in \Re^{n\times n}$ is the inertia matrix which is positive definite and symmetric, $C(q,\dot{q}) \in \Re^{n\times n}$ accounts for the centrifugal and Coriolis terms, $g(q) \in \Re^n$ is the gravity term, and $\tau \in \Re^n$ is the generalized torque acting on the links.

The equation of motion (1) has the following properties [3].

Property 1 : The matrix $N(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$is skew symmetric. Thus,

$$\dot{M}(q) = C(q,\dot{q}) + C^T(q,\dot{q}).$$

Property 2 : There exists a positive constants k_c such that

$$||C(q,\dot{q})|| \leq k_c||\dot{q}||, \qquad \forall q,\dot{q} \in \Re^n.$$

Property 3 : There exists a positive constant k_g satisfying

$$\left|\left|\frac{\partial g(q)}{\partial q}\right|\right| < k_g, \quad \forall q \in \Re^n \quad \text{and} \quad ||g(x)-g(y)|| \leq k_g||x-y||, \quad \forall x,y \in \Re^n.$$

Figure 1 shows the structure of MBDA, where P is a plant, M is a model for the plant P. The vectors $q, q_0, q_d \in \Re^n$ are a position vector of the plant, a position vector of the model, and a desired position vector respectively, $\tau, \tau_0 \in \Re^n$ are input torques for plant and model. In the figure, K, K_1, and K_2 are feedback gain matrices of appropriate dimensions. The conventional PD gains are used for K and K_1, and only D gains are used for K_2. If $K_2 = 0$, it is the same structure as in [1].

Let $\boldsymbol{q} \triangleq [q^T, \quad (q_0 - K_{p1}^{-1}g(q_d))^T]^T$, and $T \triangleq [(\tau - g(q_d))^T, \quad \tau_0^T]^T$. Then the robot dynamics of the overall system is

$$\begin{aligned} T =& \begin{bmatrix} M(q) & 0 \\ 0 & M_0(q_0) \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \ddot{q_0} \end{bmatrix} + \begin{bmatrix} C(q,\dot{q}) & 0 \\ 0 & C_0(q_0,\dot{q_0}) \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{q_0} \end{bmatrix} + \begin{bmatrix} g(q)-g(q_d) \\ 0 \end{bmatrix} \\ =& \boldsymbol{M(q)\ddot{q}} + \boldsymbol{C(q,\dot{q})\dot{q}} + \boldsymbol{g(q,q_d)}, \end{aligned} \tag{2}$$

where the subscript “0” is used to represent the terms related to the model. Note that we omit the gravity term in the model dynamics.

Assuming a constant desired position vector, i.e., $\dot{q_d}(t) = 0$ for $t > 0$, the feedback dynamics of the system in Fig. 1 becomes

$$T = -\begin{bmatrix} K_p + K_{p1} & -K_{p1} \\ K_p & 0 \end{bmatrix} \begin{bmatrix} \tilde{q} \\ \tilde{q_m} \end{bmatrix} - \begin{bmatrix} K_d + K_{d1} & -K_{d1} \\ K_d - K_{d2} & K_{d2} \end{bmatrix} \begin{bmatrix} \dot{\tilde{q}} \\ \dot{\tilde{q_0}} \end{bmatrix} = -\boldsymbol{K_p\tilde{q}} - \boldsymbol{K_d\dot{q}}. \tag{3}$$

Here, $\tilde{q} \triangleq q - q_d$, $\tilde{q_0} \triangleq q_0 - q_d$, $\tilde{q_m} \triangleq \tilde{q_0} - K_{p1}^{-1}g(q_d)$ and $\boldsymbol{\tilde{q}} \triangleq [\tilde{q}^T, \ \tilde{q_m}^T]^T$. The diagonal matrices K_p (K_{p1}) and K_d (K_{d1}) are P and D gains of K (K_1), and the diagonal matrix K_{d2} is D gain of K_2 in Fig. 1. In the above equations (2) and (3), $\boldsymbol{K_p}$, $\boldsymbol{K_d}$, $\boldsymbol{M(q)}$, $\boldsymbol{C(q,\dot{q})}$, and $\boldsymbol{g(q,q_d)}$ are appropriately defined.

From these equations, the following closed loop dynamic equation of the MBDA system in Fig. 1 is obtained:

$$\boldsymbol{M(q)\ddot{q}} + \boldsymbol{C(q,\dot{q})\dot{q}} + \boldsymbol{g(q,q_d)} = -\boldsymbol{K_p\tilde{q}} - \boldsymbol{K_d\dot{q}}. \tag{4}$$

To carry out the stability analysis, we consider the following candidate Liapunov function:

$$V = \frac{1}{2}(\tilde{\boldsymbol{q}}^T \boldsymbol{K} \tilde{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \boldsymbol{M} \dot{\boldsymbol{q}}) + \frac{1}{\gamma}\tilde{\boldsymbol{q}}^T \boldsymbol{M} \dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{x}^T \begin{bmatrix} \boldsymbol{K} & \frac{1}{\gamma}\boldsymbol{M} \\ \frac{1}{\gamma}\boldsymbol{M} & \boldsymbol{M} \end{bmatrix} \boldsymbol{x} \triangleq \boldsymbol{x}^T \boldsymbol{L} \boldsymbol{x}, \tag{5}$$

where $\boldsymbol{K}$ is a symmetric positive definite constant matrix and γ is a positive constant and $\boldsymbol{x} \triangleq [\tilde{\boldsymbol{q}}^T, \dot{\boldsymbol{q}}^T]^T$.

Using the following inequality

$$2||u^T A v|| \leq a_1 u^T A u + \tfrac{1}{a_1} v^T A v, \qquad a_1 > 0 \tag{6}$$

which holds for any positive definite matrix $A \in \Re^{n\times n}$ and for any vector $u, v \in \Re^n$, V can be shown to be a valid Liapunov function such that it becomes positive definite

$$V \geq \frac{1}{2}\tilde{\boldsymbol{q}}^T(\boldsymbol{K} - \frac{a_1}{\gamma}\boldsymbol{M})\tilde{\boldsymbol{q}} + \frac{1}{2}(1 - \frac{1}{a_1\gamma})\dot{\boldsymbol{q}}^T \boldsymbol{M} \dot{\boldsymbol{q}} \geq 0.$$

if for an arbitrary positive constant a_1, the following condition holds

$$a_1\gamma > 1, \quad \boldsymbol{K} - \frac{a_1}{\gamma}\boldsymbol{M} > 0. \tag{7}$$

Theorem: Let $K_p = K_{p1}, \quad K_d = K_{d1} + K_{d2}$ and

$$\boldsymbol{K} = \begin{bmatrix} 2K_p + \frac{1}{\gamma}(K_d + K_{d1}) & -K_p + \frac{1}{\gamma}K_{d1}) \\ -K_p + \frac{1}{\gamma}K_{d1} & \frac{1}{\gamma}K_{d2} \end{bmatrix}.$$

Also let $\Omega \triangleq \{\boldsymbol{x} : ||\boldsymbol{x}|| < b\sqrt{\underline{\lambda}/\overline{\lambda}}\}$, where $\underline{\lambda}$ and $\overline{\lambda}$ are the minimum and maximum eigenvalues of $\boldsymbol{L}$ respectively.

For a constant input q_d, the system in Fig.1 is asymptotically stable at the origin $\boldsymbol{x}_0 = \boldsymbol{0}$ and Ω is a region of asymptotic stability, if the following conditions are satisfied.

$$\begin{aligned} &K_d + K_{d1} - \frac{1}{\gamma}M - \frac{k_c b}{\gamma}I_n - \frac{a_3 k_g}{2}I_n > 0 \\ &K_{d2} - \frac{1}{\gamma}M_0 - \frac{k_{c0} b}{\gamma}I_n - a_2 E > 0 \\ &2K_p - k_g I_n - \frac{\gamma}{a_2}E - \frac{\gamma k_g}{2a_3}I_n > 0 \end{aligned} \tag{8}$$

Here a_2, a_3 and b are arbitrary positive constants, I_n is an $n \times n$ identity matrix, and the elements of matrix E is given as $E_{ij} = \begin{cases} |-K_{pij} + \frac{1}{\gamma}K_{d1ij}| & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j. \end{cases}$

The constant k_{c0} is chosen to meet the *Property 2* for model dynamics such that $||C_0(q_0,\dot{q}_0)|| \le k_{c0}||\dot{q}_0||$.

Proof: Differentiating (5), $\dot{V}$ becomes

$$\begin{aligned}
\dot{V} =& \frac{1}{2}(2\dot{\boldsymbol{q}}^T(\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{K}\tilde{\boldsymbol{q}})+\dot{\boldsymbol{q}}^T\dot{\boldsymbol{M}}\dot{\boldsymbol{q}})+\frac{1}{\gamma}(\dot{\boldsymbol{q}}^T\boldsymbol{M}\dot{\boldsymbol{q}}+\tilde{\boldsymbol{q}}^T\dot{\boldsymbol{M}}\dot{\boldsymbol{q}}+\tilde{\boldsymbol{q}}^T\boldsymbol{M}\ddot{\boldsymbol{q}})\\
=& \dot{\boldsymbol{q}}^T(\boldsymbol{K}-\boldsymbol{K_p}-\frac{1}{\gamma}\boldsymbol{K_d}^T)\tilde{\boldsymbol{q}}-\dot{\boldsymbol{q}}^T(\boldsymbol{K_d}-\frac{1}{\gamma}\boldsymbol{M})\dot{\boldsymbol{q}}-\dot{\boldsymbol{q}}^T\boldsymbol{g(q,q_d)}-\frac{1}{\gamma}(\tilde{\boldsymbol{q}}^T\boldsymbol{K_p}\tilde{\boldsymbol{q}}-\tilde{\boldsymbol{q}}^T\boldsymbol{C}^T\dot{\boldsymbol{q}}+\tilde{\boldsymbol{q}}^T\boldsymbol{g(q,q_d)})\\
=& 2\dot{q_0}^T(-K_p+\frac{1}{\gamma}K_{d1})\tilde{q}-\dot{q}^T(K_d+K_{d1}-\frac{1}{\gamma}M)\dot{q}+\frac{1}{\gamma}\tilde{q}^TC^T\dot{q}-\dot{q_0}^T(K_{d2}-\frac{1}{\gamma}M_0)\dot{q}_0\\
&+\frac{1}{\gamma}\tilde{q_m}^TC_0^T\dot{q}_0-\frac{2}{\gamma}\tilde{q}^TK_p\tilde{q}-\dot{q}^T\{g(q)-g(q_d)\}-\frac{1}{\gamma}\tilde{q}^T\{g(q)-g(q_d)\}.
\end{aligned}$$

The second equality is from *Properties 1* and (4) and the third equality is obtained using $K_p = K_{p1}$, $K_d = K_{d1}+K_{d2}$ and the definition of $\boldsymbol{K}$. Using *Property 2* and *3*, the terms related to the Corioris and gravity forces are bounded such that

$$\begin{aligned}
&||\tilde{q}^TC^T\dot{q}|| \le k_c||\dot{q}||^2||\tilde{q}||, \quad ||\tilde{q_m}^TC_0^T\dot{q}_0|| \le k_{c0}||\dot{q}_0||^2||\tilde{q_m}||\\
&||\dot{q}^T(g(q)-g(q_d))|| < k_g||\dot{q}||||q-q_d|| = k_g||\dot{q}||||\tilde{q}||\\
&||\tilde{q}^T(g(q)-g(q_d))|| < k_g||\tilde{q}||||q-q_d|| = k_g||\tilde{q}||^2,
\end{aligned} \tag{9}$$

and this leads to

$$\begin{aligned}
\dot{V} \le\ & 2\dot{q_0}^T(-K_p+\frac{1}{\gamma}K_{d1})\tilde{q}-\dot{q}^T(K_d+K_{d1}-\frac{1}{\gamma}M-\frac{k_c||\tilde{q}||}{\gamma}I_n)\dot{q}\\
&-\dot{q_0}^T(K_{d2}-\frac{1}{\gamma}M_0-\frac{k_{c0}||\tilde{q_m}||}{\gamma}I_n)\dot{q}_0-\frac{2}{\gamma}\tilde{q}^TK_p\tilde{q}+k_g||\dot{q}||||\tilde{q}||+\frac{1}{\gamma}k_g||\tilde{q}||^2\\
\le& -\dot{q}^T(K_d+K_{d1}-\frac{1}{\gamma}M-\frac{k_c||\tilde{q}||}{\gamma}I_n-\frac{a_3k_g}{2}I_n)\dot{q}-\dot{q_0}^T(K_{d2}-\frac{1}{\gamma}M_0-\frac{k_{c0}||\tilde{q_m}||}{\gamma}I_n-a_2E)\dot{q}_0\\
&-\frac{1}{\gamma}\tilde{q}^T(2K_p-k_gI_n-\frac{\gamma}{a_2}E-\frac{\gamma k_g}{2a_3}I_n)\tilde{q}.
\end{aligned}$$

The second inequality is by applying (6) to every non-quadratic terms. Note that if condition (8) holds, $\dot{V}$ becomes negative semi-definite for all the points within $\Gamma = \{\boldsymbol{x} : ||\boldsymbol{x}|| < b\}$.

Because $\underline{\lambda}||\boldsymbol{x}||^2 \le V \le \overline{\lambda}||\boldsymbol{x}||^2$, the path of $\boldsymbol{x}$ starting on Ω will not leave the region Γ. The necessary condition for $\dot{V} = 0$ is $\boldsymbol{x}^T = [\tilde{q},\tilde{q}_m,\dot{q},\dot{q}_0] = [0,\tilde{q}_m,0,0]$. For this point to be stable, $\tilde{q}_m$ must be zero, because from (4) it becomes $M\ddot{q}+C\dot{q}+g(q) = -(K_p+K_{p1})\tilde{q}-(K_d+K_{d1})\dot{q}+K_{p1}\tilde{q}_0+K_{d1}\dot{q}_0$. Thus the origin $\boldsymbol{x}_0 = \boldsymbol{0}$ is the only point contained in the largest invariant set in $R = \{\boldsymbol{x} : \boldsymbol{x} \in \Omega, \dot{V} = 0\}$. Finally, using the Theorem VI of [4], the origin is asymptotically stable and every point in Ω tends to the origin as $t \to \infty$. $\square$

Although the conditions in (8) seem hard to be satisfied, these conditions can be easily met if K_p, K_{d1}, and K_{d2} can be set sufficiently large. Note that at the equilibrium point, the position error of the plant is $\tilde{q} = 0$, but the position error of the model becomes $\tilde{q}_0 = K_{p1}^{-1} g(q_d)$. The gravitational force $g(q)$, which is a disturbance, is completely compensated by the position error of the model and do not affect the plant output in the steady state, because the model feedback gain K_2 does not contain a proportional component.

In many robot manipulators, not only the Coriolis/centrifugal forces but also the inertia matrix is not easy to estimate. For this case, we can model a robot with a constant inertia matrix M_0 using only its diagonal components and set $C_0 = 0$. This simplifies the modelling process greatly without degrading the performance. A simulation result for a two-link robot with this method was presented in [2].

III. Conclusions

In this letter, a new method for controlling robot manipulators is proposed and its stability is proved. The proposed method is easy to implement and very robust in regard to modelling errors and disturbances. It consists of a model in parallel with the plant. In the presence of disturbances, this method attenuates the disturbance significantly. This MBDA controller has both the advantage of a PD controller in that it is asymptotically stable and the advantage of a PID controller which can eliminate steady state errors due to modelling errors or disturbances.

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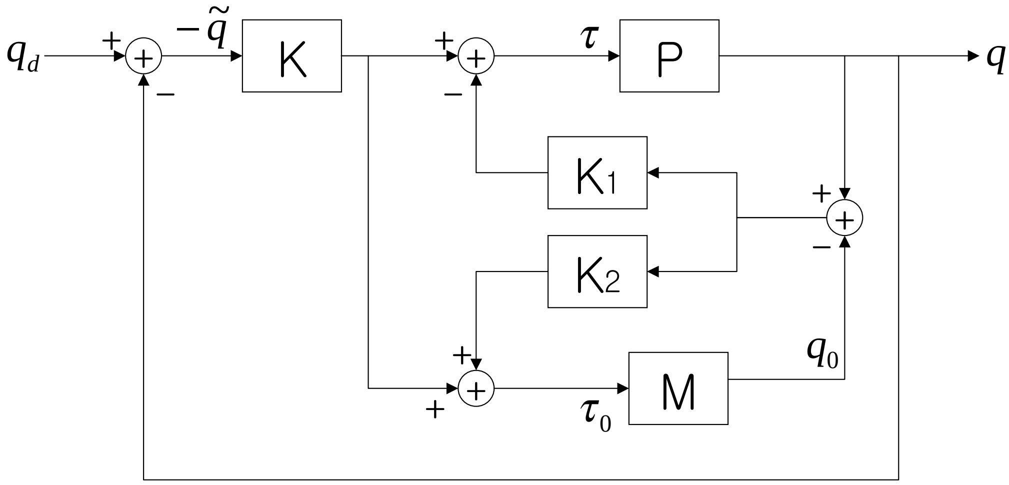

Fig. 1. MBDA Controller