Robust face recognition in a polar coordinate system using LDAr

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Abstract

In this paper, we propose a novel method for face recognition which uses a polar coordinate system instead of the conventional cartesian coordinate system. The polar coordinate system delineates near-pole area more vividly than the area far from the pole. Therefore, a polar face image can achieve more vivid representation of the important central facial regions compared to the conventional cartesian face image. To cope with a small amount of rotation and scaling change in a frontal face image, we applied a method based on regressional version of linear discriminant analysis (LDAr). The experimental results show that the proposed method achieves a good recognition performance under varying rotation and scaling.

Keywords: Face recognition, rotation, scaling, polar coordinate system, LDAr

1. Introduction

Recently, face recognition under ambient and unfavorable conditions has been studied extensively in computer vision and pattern recognition community [1]. In face recognition, a face image can be represented as a set of individual pixels and feature extraction is very important. In doing so, dimensionality reduction methods are typically used to reduce the number of input variables to simplify the problems without degrading performances [2]. With dimensionality reduction methods, we can handle feature vectors more effectively and can solve the curse of dimensionality which occurs when the input dimension is huge. Among these, LDA is successfully used to find a projection, from the original space to a lower dimensional space, which maximizes the between-class scatter while minimizing the within-class scatter [3]. Most face recognition method extract features in the original image using conventional cartesian coordinate. On the other hand, polar coordinate transform is used in iris recognition [4] and object recognition [5]. In a face, important regions such as eyes, nose and mouth are concentrated on the central part of a face and polar coordinate system is more suited to represent these characteristics of a frontal face image [6]. Therefore, in this paper, polar coordinate system is used instead of the conventional cartesian coordinate system to improve recognition rate as in [6].

To cope with a small amount of rotation and scaling change in a frontal face image, we applied a method based on regressional version of linear discriminant analysis [7] which tries to maximize the ratio of distances of samples with large differences in target value and those with small differences in target value to polar face

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images.

In Section 2.1, the polar coordinate transform for face recognition is introduced. Section 2.2 addresses the concept of LDA for regression (LDAr). Section 2.3 presents the methods to cope with the unfavorable conditions such as rotation and scaling variations based on LDAr. The experimental results and analysis are shown in Section 3 and Section 4 concludes the paper with a mention of future works.

2. Methods

2.1 Polar coordinate system

In transforming cartesian image to a polar image, among the central area of a face, we select a point as a pole and make a polar image of a face by evenly sampling pixels in each direction of 360 degrees around the pole. In a face, important regions such as eyes, nose and mouth are concentrated on the central part of a face and it is assumed that these feature points play more important roles in discriminating individuals than the other areas. The polar coordinate system naturally achieves this purpose because in a polar coordinate system, the near-pole area is more vividly represented than the area far from the pole. It was shown that the classification performances of polar images are better than those of cartesian images in [6].

Fig. 1(a) shows a face image in 80 × 100 Yale database and Fig. 1(b) shows the corresponding pole and orientation of the polar coordinate image with the radius of 40 pixels.

(a)

(b)
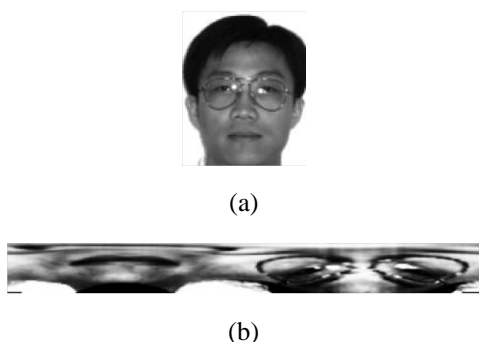
Fig. 1. The original face image (a) and the transformation of cartesian to polar image with radius 40 (b)

In the transformation, the location of a pole is crucial and in our experiments, we manually marked the location of the tip of a nose in each image and the average location was used as the pole. The resultant location was (x, y)=(40, 65) for Yale database [6]. The bi-cubic interpolation is used for the transformation and the intensity of the points outside the cartesian image is set to 0 (black).

2.2 LDA for regression (LDAr)

In the classification problem, LDA is one of the well known methods for supervised dimensionality reduction [3]. LDA finds projective directions by maximizing the ratio of between-class scatter matrix to within-class scatter matrix. In [7], this idea was extended to solve regression problems and a new feature extraction method LDAr for regression problems was proposed. Consider a set of input/target pairs $\{x_i, y_i\}_{i=1}^n$. In this regression setting, we are to find a set of features f_i's ($= w_i^T x$), which are linear transformations of x such that they contain much information about the target variable y. To achieve this goal, Fisher's criterion was modified appropriately to fit in the regression problems in [7].

The classification problems have discrete target variables, or classes. Compare to classification problem, in regression problems it is difficult to define between-class scatter and within-class scatter because target variables, classes are continuous.

In [7], the simple concept that the samples with small differences in the target variables can be considered as belonging to the same class and the ones with large differences in the target variables can be considered as belonging to the different classes was applied. By this idea of soft class, the between-scatter matrix and within-scatter matrix in LDA were modified as:

$$S_{br} = \frac{1}{n_b} \sum_{(i,j) \in A_{br}} f(y_i - y_j)(x_i - x_j)(x_i - x_j)^T$$

$$S_{wr} = \frac{1}{n_w} \sum_{(i,j) \in A_{wr}} f(y_i - y_j)(x_i - x_j)(x_i - x_j)^T$$

where

$A_{br} = \{(i,j) \mid \mid y_i - y_j \mid \geq \tau, i \prec j\}$ and

$A_{wr} = \{(i,j) \mid \mid y_i - y_j \mid \prec \tau, i \prec j\}$.

The function $f(\square)$ is a weight function which takes on a positive value and τ is a threshold separating an index pair into A_{br} and A_{wr} and $n_b = \mid A_{br} \mid$, $n_w = \mid A_{wr} \mid$.

The threshold τ can be represented as a multiple of the standard deviation of the target variable ($\tau = \alpha\sigma_y$).

Finally, Fisher's criterion can be modified as

$$W = \arg\max_W \frac{\mid W^T S_{br} W \mid}{\mid W^T S_{wr} W \mid}$$

Maximizing the above Fisher's criterion is equvalent to solving the following eigenvalue decomposion problem:

$$S_{wr}^{-1} S_{br} w_k = \lambda_k w_k \qquad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$$

And linear projections w_k's can be found.

2.3 Coping with rotation and scaling

In this paper, we estimate the amount of rotation and scaling factor of polar face image by LDAr. **Fig. 2** shows a flowchart of regression process using LDAr. The training data and test data consist of polar images which are obtained by rotating and scaling original images by a random amount in a polar coordinate system.

Firstly, we normalize the training data so as to make each pixel have zero mean and unit variance. Then, PCA is performed to project the higher-dimensional image space onto a lower dimensional sub-space. After PCA, LDAr is applied. In LDAr we first calculate the between-class scatter S_{br} and the within-class scatter S_{wr}. Then we calculate the eigenvectors corresponding to the set of the largest eigenvalues and use them to form the most discriminant projection vector set for LDAr. Finally, the projected samples of training data and testing data are obtained and regression is performed on these datasets using 3-nearest neighbor algorithm as in [7].

In our experiment, original images were randomly rotated up to 20°, 30°, 40° and scaled with scaling factors of 0.8, 0.9, 1.0, 1.1 and 1.2. In LDAr, we set α = 0.3 and weight function $f(x) = \sqrt{\mid \mid x \mid - \tau \mid}$ was used.
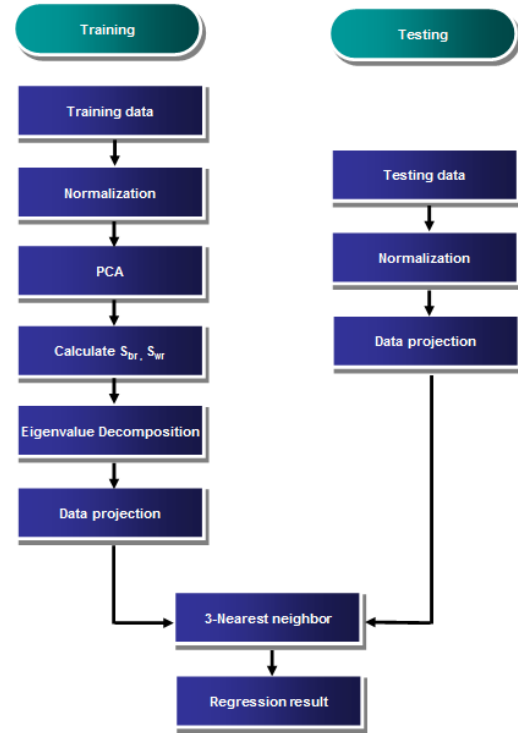
Fig. 2. Flowchart of regression process using LDAr.

3. Experimental Results and Analysis

In this section, the accuracy of the proposed algorithm is evaluated experimentally on Yale database [8].

The Yale database consists of 165 images which contain 15 individuals and 11 images per each person. Original images in cartesian coordinate are 100x80 pixels in size with gray color.
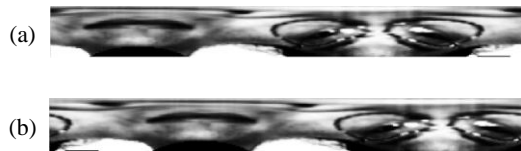
A polar image rotated by +22° and -26° clockwise are shown in **Fig. 3** and the ones scaled with scaling factors of 0.8, 0.9, 1.0, 1.1 and 1.2 are shown in **Fig. 4**. For all the experiments, the size of each polar image is 40x360 pixels, with 256 grey levels per pixel.

The Experimental results using LDAr under various rotation angles are reported in **Table 1**. In the experiment, we rotated each of the 165 polar Yale images up to 20°, 30° and 40° with uniform random distribution and this procedure was repeated 10 times per each image to obtain total 1650 (=165x10) images. Among 1650 images, 770 images were randomly selected for
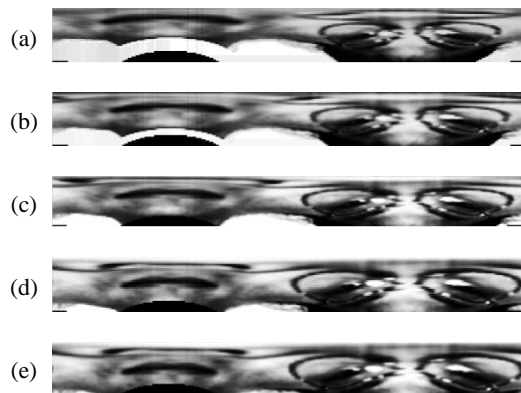
training and the remaining 880 images were used for testing. In the table, we can see that the maximum rms error for test data is 2.4581 which is relatively small.

Table 2 shows the experimental results obtained using LDAr under various scaling factors. In the table, we show the rms error of training and test data. For classification, we chose the 3-nearest neighbor (3-NN) algorithm. We set the target variable as 8, 9, 10, 11 and 12. The target variable 8 means we scaled original image with scaling factor 0.8. We scaled each original 165 Yale database images in above scaling factor. Among total 825 (=165x5) imges, 385 imges were used for training and the remaining ones were used for testing. The results show that the training rms error is 0.0000 and testing rms error is 0.4356.

Table 3 shows the experimental results obtained using LDAr under varying scaling and rotation simultaneously. In the table, we show the training rms error and testing rms error for polar images. We rotated each original images up to 20° and this procedure was repeated 10 times per each image.

Then, the total 1650 (=165x10) images were scaled with a unifom random scaling factors of 0.8, 0.9, 1.0, 1.1 and 1.2. Among 1650 images, 770 randomly selected images were used for training and the remaining 880 images were used for testing.

Table 1. Experimental results of LDAr under various rotation angles

Only rotation variation	±20°	±30°	±40°
Tr rms error (°)	0.0000	0.0297	0.0572
Test rms error (°)	2.4581	1.4209	1.6588

Table 2. Experimental results of LDAr under various scaling factors

Only scaling variation	Scaling factor (0.8~1.2)
Tr rms error	0.0000
Test rms error	0.4356

(a)

(b)

Fig. 3. A polar image rotated with +22°(a), -26° (b)

(a)

(b)

(c)

(d)

(e)

Fig. 4. A polar image scaled with scaling factors 0.8(a), 0.9(b), 1.0(c), 1.1(d) and 1.2(e)

Table 3. Experimental results of LDAr under varying scaling factor and rotation angle simultaneously

Scaling & rotation variation	Error in rotation angle		
	0.8~1.2 -20°~+20°	0.8~1.2 -30°~+30°	0.8~1.2 -40°~+40°
Tr rms error (°)	0.0528	0.0360	0.0710
Test rms error (°)	1.4440	1.4116	1.6853

Scaling & rotation variation	Error in scaling factor		
	0.8~1.2 -20°~+20°	0.8~1.2 -30°~+30°	0.8~1.2 -40°~+40°
Tr rms error	0.0000	0.0000	0.0000
Test rms error	0.7545	0.5600	0.5973

In the same way, we also rotated each images up to 30° and 40° while scaled rotated images with same scaling factors.

From **Table 3**, we can see that the testing rms error is below 1.6853 ° when we estimate rotation degree. The result is almost the same as the one in **Table 1** which was performed without varying scaling factor. Regarding the estimation of the scaling factors, the errors in **Table 3** are somewhat bigger than those in **Table 2**.

4. Conclusions

This paper addresses a method for face recognition which uses a polar coordinate system instead of the conventional cartesian coordinate system. The polar coordinate system delineates near-pole area more vividly than the area far from the pole. To cope with a small amount of rotation and scaling change in a frontal face image, we applied a method based on regressional version of linear discriminant analysis which tries to maximize the ratio of distances of samples with large differences in target value and those with small differences in target value to polar coordinate system. The proposed methods performed quite satisfactorily for Yale Database.

The limitations of the proposed methods are *1) they only apply to frontal face images* and *2) they are susceptible to mis-location of the pole*. Besides improving the proposed methods, we would like to tackle these problems in the future.

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